SEGMENTATION OF POLARIMETRIC SAR DATA WITH A MULTI-TEXTURE PRODUCT MODEL

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ABSTRACT

The previously proposed multi-texture model for multi-looked PolSAR data statistics [1] is hereby implemented into an advanced statistical clustering algorithm and tested on several real PolSAR images. The multi-texture model is based on the product model for SAR statistics, yet allows the possibility of different texture parameters for the co-polarized (co-pol) and cross-polarized (cross-pol) channels. The implementation automatically determines the most appropriate texture model between the proposed “dual-texture” model and the traditional “scalar-texture” model. The clustering algorithm is implemented as a multi-texture version of [2]. It incorporates the flexible U-distribution, contextual smoothing with Markov random fields, and determines the number of classes with goodness-of-fit tests. The real SAR examples indicate that multi-texture is not generally required and we discuss the possible mis-interpretation of multi-texture in alternative window-based estimation methods, due to mixing of different polarimetric classes.

1. INTRODUCTION

The multivariate product model with scalar texture has been widely used in non-Gaussian modeling of polarimetric data (e.g. [3]). This model states that the backscattered vector signal results from the product between a Gaussian speckle noise vector component and a positive, random scalar texture component. Hence, the assumption is that the texture has the same statistical distribution and is fully correlated in all polarimetric channels. In a recent paper [1], the authors examined an extended multivariate product model, in which each polarimetric channel was characterized by an individual random texture variable. This model extension included development of probability density functions, log-cumulant expressions and validation hypothesis tests. The preliminary analysis on several distinctly different PolSAR scenes revealed that in some cases the data supported a dual or two-component texture model, with one texture variable associated with the co-pol and another with the cross-pol channels. No evidence was however found in support of the more general four-component texture model.

In the current paper, we take the dual-texture theory and implement it into an image clustering algorithm. The idea is simple - any data classes that exhibit different textural variation in the different channels should obtain better results under the dual-texture model. An existing flexible U-distribution-MRF model [2] has been adapted to support scalar or dual-texture parameters, and chooses the best texture model automatically. The clustering algorithm combines the multi-texture class model with the flexible U-distribution for pixel-wise variation, a Markov random field (MRF) model for contextual smoothing, and goodness-of-fit testing to optimize the segmentation and the determination of the appropriate number of classes.

The performance of the algorithm is first tested on simulated PolSAR data and confirms the model choice and parameter estimation abilities. However, the results for several real PolSAR data-sets does not find any evidence for the dual-texture case, that is, the scalar-texture model is chosen for all classes. We believe that the previously found evidence for multi-texture, may actually have been caused by mixing pixels of different classes within the estimation window. This situation is avoided by doing pixel-wise clustering, where these dissimilar pixels are segmented and apparent dual-texture caused by class mixtures is eliminated by the goodness-of-fit tests. Less sophisticated algorithms that do not verify the model fit of the clusters may end up with mixtures that appear to have dual-texture.

The multi-texture model theory is summarized in section 2, the clustering algorithm implementation is described in section 3, some results are shown in section 4 with some discussion, and we finish with our conclusions.

2. MULTI-TEXTURE PROBABILITY DENSITY FUNCTION FOR MULTI-LOOKED POLSAR DATA

Let the scattering vector be given as

\[ s = [s_{hh}; s_{hv}; s_{vh}; s_{vv}]^T, \]

where the indices refer to the polarization of received and transmitted electromagnetic waves, respectively. In the multi-texture statistical model \( s \) is formulated as

\[ s = T^{1/2}x \]  \hspace{1cm} (1)

where \( T = \text{diag}\{t_{hh}; t_{hv}; t_{vh}; t_{vv}\} \) is a diagonal matrix containing the texture variables associated with the respective polarized channels, \( T^{1/2} \) denotes its matrix square root, and \( x \) is
a zero mean, circular complex Gaussian vector variable representing speckle [4]. Hence, the covariance of $s$, conditioned on the texture matrix $T$, i.e. $\Sigma_s|T$, is

$$\Sigma_s|T = T^{1/2} \Sigma_x T^{1/2}$$  \hspace{1cm} (2)

with $\Sigma_x$ as the covariance of $x$. It is well-known that the probability density function (pdf) of the sample covariance matrix for a Gaussian vector follows the scaled Wishart distribution [5]. If the sample covariance of $x$ is denoted $W$, the sample covariance matrix of $s$, denoted $C$, becomes

$$C = \frac{1}{L} \sum_{i=1}^{L} s_i s_i^H = T^{1/2} W T^{1/2}$$  \hspace{1cm} (3)

with $(\cdot)^H$ as the Hermitian transpose. Hence, the pdf of $C$ given $T$, is obtained by transformation of variables to become

$$f_{C|T}(C|T; L, \Sigma_x) = \frac{L^{Ld} \cdot |C|^{L-d} \cdot \text{etr}(-L\Sigma^{-1}_x T^{-1} C T^{-1})}{\Gamma_d(L) |T|^d |\Sigma_x|^d}$$  \hspace{1cm} (4)

where $J_{W\rightarrow C}$ is the Jacobian of the transformation from $W$ to $C$. The marginal distribution for $C$ is obtained by integrating over the pdf of $T$, i.e.

$$f_C(C; L, \Sigma_x) = \int f_{C|T}(C|T; L, \Sigma_x) f_T(T) dT.$$  \hspace{1cm} (5)

If we consider backscatter geometry, reciprocity and reflection symmetry, and in addition make the assumption that the texture components $t_{hh}$ and $t_{vv}$ are equal, the distribution of the sample covariance matrix $C$ was shown in [1] to be given by (6), where $q_i$ denotes the $i$th element of $\Sigma^{-1}_x$, and $f_{t_{hh}}(t_{hh})$ and $f_{t_{vv}}(t_{vv})$ denotes the probability density functions of $t_{hh}$ and $t_{vv}$, respectively. The model proposed in (6) gives the freedom to assume different texture distributions for the co- and cross-pol channels. The integrals can be evaluated in closed form for texture models like the gamma, the inverse gamma, the normal inverse Gaussian, and the Fisher distributions, to name a few.

### 3. IMPLEMENTATION AND ANALYSIS

In the proposed image segmentation, pixels are separated into clusters based upon the either the traditional scalar-texture model or the multi-texture model in (6). The statistical approach for clustering the images uses the iterative expectation maximization algorithm with a few modifications, as has been described in detail in [6] and [2]. The key features are:

(a) flexible non-Gaussian distributions  
(b) fast parameter estimation  
(c) contextual-smoothing  
(d) automatic number of classes

The $U$-distribution was chosen to model the PolSAR statistics because it is the most flexible of the commonly used models and includes the others as special cases, that is, the Wishart, K-Wishart and $G^0$ models. Parameter estimation is achieved within the algorithm by the method of matrix log-cumulants (MoMLC), which is fast to compute and achieves the most accurate results [5]. Contextual smoothing is implemented using a Markov Random Field modeling of the class labels. In this way, the algorithm gives more weight to the class memberships of spatially neighboring classes. The class label MRF easily combines with clustering based on a finite mixture model for the pixel covariance matrix distributions, by replacing the global class prior probabilities with spatially varying local prior probabilities determined from the local neighborhoods.

The new extension proposed here, is that each class is modeled with either a scalar or a dual-component texture model and that this choice is determined automatically, each iteration, by an appropriate hypothesis test. The test first estimates texture parameters from both the full matrix, for the scalar-texture model, and from separated co-pol and cross-pol channels, for the proposed dual-texture case. The matrix log-cumulants are then calculated for each case and a distance measure based on multiple log-cumulants determines the best (closest) model. Subsequent pdf evaluation uses the chosen texture case and its parameters.

### 4. RESULTS AND DISCUSSION

The clustering algorithm appears to work as planned and a simulated two class test image was almost perfectly segmented. One class was generated as a scalar-texture K-Wishart model, and the other class was generated as a dual-texture model, with co-pol K-Wishart and cross-pol $G^0$ models. The generated and estimated model parameters are shown in Table 1. The algorithm automatically detected two classes and that one class displayed dual-texture. The segmented result, class histograms and log-cumulant diagrams are shown in Fig. 1. In the histograms (c), the peak width is a visual

<table>
<thead>
<tr>
<th>Class 1</th>
<th>K-Wishart $\alpha = 15$</th>
<th>U-distribution $\alpha = 16.5, \lambda = 4220$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 2</td>
<td>Co-pol</td>
<td>$\alpha = 10.4, \lambda = 217$</td>
</tr>
<tr>
<td></td>
<td>K-Wishart $\alpha = 10$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cross-pol $\lambda = 30$</td>
<td>$\alpha = 4220, \lambda = 28.8$</td>
</tr>
</tbody>
</table>
$$f_c(C; L, \Sigma_x) = \frac{L^{3L}}{\Gamma_3(L)} \frac{|C|^{L-3}}{|\Sigma_x|^L} \times \int \frac{1}{t_{hh}^L} \exp \left(-L \left(\frac{q_{11}c_{11} + q_{13}c_{31} + q_{31}c_{13} + q_{33}c_{33}}{t_{hh}}\right)\right) f_{t_{hh}}(t_{hh}) \, dt_{hh}$$
$$\times \int \frac{1}{t_{hv}^L} \exp \left(-L \left(\frac{q_{22}c_{22}}{t_{hv}}\right)\right) f_{t_{hv}}(t_{hv}) \, dt_{hv}$$

Fig. 1. Results for simulated two-class test image. (a) Simulated data representation, (b) Segmented result, (c) Class histograms, dual-texture (top) and scalar-texture (bottom), (d) Total class log-cumulant diagram, and (e) Dual-texture class individual log-cumulants.

Fig. 2. San Francisco City results, Radarsat-2, 25-looks. Class histograms, Pauli RGB and segmented image.

measure of the degree of texture, and two histograms are plotted for class 1 (top) because it was found to be the dual-texture case. The log-cumulant diagram (d) shows the total log-cumulant estimate and model fit with 95% confidence ellipse for each class, and diagram (e) shows the log-cumulants for the co-pol and cross-pol channels for class 1. Several other parameter choices were tested and all indications are that the algorithm works well and the estimated values are good, which is very encouraging.

Several real data-sets were tested and the algorithm produced reasonable segmented results, that is, the clustering results look very good under visual comparison to the Pauli RGB images. However, the real images tested do not reveal the dual-texture model, i.e., the scalar-texture model is chosen in all cases. This is even for images, such as San Francisco City and the Amazon rainforest, where we have previously "found" the dual-texture case [1] and [in prep.]. The first example is shown in Fig. 2 for San Francisco, is a Radarsat-2 sample image from 9 April, 2008. The second in Fig. 3 for the Amazon rainforest, ALOS PALSAR sample data from 13 March, 2007. They display the Pauli RGB image, the segmented result, and the class histograms. As may be seen, all class histograms are for the scalar texture case (one plot each) and are (mostly) reasonable fits to the data histograms producing visually satisfactory segmented images.

These images have been extensively tested under several multi-look averaging sizes and different degrees of sensitivity, via sub-sampling, plus other tuning options. Our conclusion is that the manually chosen box-window method previously used to estimate texture had mis-interpreted pixels from a mixture of classes as the dual-textured case.

This is understandable because class mixtures often measure a significantly high texture parameter, but it is actually due to mixtures of different brightnesses. One can easily visualize that mixtures with different polarimetry could cause the appearance of multi-texture. Consider two mixture classes with markedly different polarimetry, such that the brightness difference between classes is different for the co-pol and cross-pol channels. Then the texture measured for the co-pol mixture may have a different estimated texture than the cross-
5. CONCLUSIONS

We have developed and tested a clustering algorithm that incorporates both the scalar and dual-texture models for PolSAR image analysis. The flexible $U$-distribution with Markov random field contextual smoothing and goodness-of-fit testing was used for the demonstration. The algorithm was proven on simulated test data with excellent results. Several real datasets were satisfactorily segmented with the automatic algorithm, as judged by visual inspection and histogram analysis. However, none of the datasets tested showed any evidence for the dual-texture case. We explain that window-based estimation methods may inadvertently measure mixtures of classes, which can result in an effect similar to multi-texture. This is actually from poorly fitting models to mixed histograms and not ingralned within the pixel level data as supposed by multi-texture theory. The pixel level clustering algorithm is actually separating these mixed pixels into classes that only exhibit the scalar-texture case. This means that the less complicated scalar-product model is generally suitable of PolSAR analysis.

6. REFERENCES


