OCEAN CLUTTER MODELING FOR SHIP DETECTION

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ABSTRACT

This work addresses the problem of covariance matrix estimation for ocean clutter modeling. For ship detection based on polarimetric synthetic aperture radar (PolSAR) imagery and constant false alarm rate (CFAR) detectors, accurate ocean clutter modeling is essential. The covariance matrix provides all the polarimetric information of the ocean clutter and its estimate is always involved in PolSAR detection [1]. The aim of this work is to investigate and compare the behavior of different covariance matrix estimators, i.e., the sample mean, fixed-point, and maximum likelihood estimators. An approximate maximum likelihood covariance matrix estimator is also proposed and discussed for better computational efficiency. Their performances are evaluated in terms of the Kullback-Leibler (KL) matrix distance, and computational efficiency. Various textured ocean clutter conditions are considered, ranging from high texture to the non-textured case with Gaussian clutter. Experiments are performed on simulated ocean clutter data.

Key words: Ocean Clutter Modeling; Covariance Matrix Estimation; Synthetic Aperture Radar.

1. INTRODUCTION

Polarimetric synthetic aperture radar (PolSAR) is a well-known sensor for remote sensing applications, which is capable of producing high quality images of the Earth. Ship detection based on PolSAR imagery can be achieved with a constant false alarm rate (CFAR) detector. However, accurate statistical modeling of the ocean clutter is essential to meet the specified false alarm rate. Many studies have focussed on the statistical analysis and modeling of ocean PolSAR images. One commonly applied polarimetric ocean clutter model is the product model [2,3], which is considered to be an appropriate statistical model for ocean clutter in various scenarios. The radar backscattering is modeled as a product between the square root of an random variable (texture) and an independent zero-mean circular complex Gaussian random vector (speckle). When the texture random variable is assumed to be gamma distributed, the product model becomes the complex multivariate K-distribution.

Most natural scenes (e.g., ocean clutter) are considered as distributed scatterers, therefore all their polarimetric information are provided in the covariance matrix [4]. For polarimetric radar detection, covariance matrix estimation is always considered to be an important factor [1]. Covariance matrix estimates are applied in many applications, for instance, the polarimetric whitening filter (PWF) introduced by Novak et al. [5,6]. PWF processes the single look complex (SLC) polarimetric scattering vector into full-resolution pixel intensity and provides effective speckle reduction, which may also lead to enhanced target detection performance.

The main objective of this work is to investigate and compare the behavior of different existing covariance matrix estimators: the sample mean, fixed-point, and maximum likelihood estimators. In addition, an approximation of the maximum likelihood covariance matrix estimator is introduced for better computational efficiency. This work is based on simulated ocean clutter, with a population mean covariance matrix obtained from real ocean clutter measurements in a Radarsat-2 quadrature polarimetric (quad-pol) SAR dataset.

This paper is organized as follows. Section 2 provides a brief introduction of PolSAR data statistics. Section 3 is devoted to give detailed descriptions of the sample mean, the fixed-point, and the maximum likelihood covariance matrix estimators. An approximate maximum likelihood covariance matrix estimator is proposed based on the lower and upper bounds for a ratio of Bessel K functions [7]. In Section 4, different covariance matrix estimators are examined under various texture conditions, ranging from high texture to the non-textured case with Gaussian clutter. Their performance is evaluated with the Kullback-Leibler (KL) matrix distance and by computational efficiency. Advantages and limitations are also discussed. Eventually, Section 5 presents the main conclusions and perspectives derived from the work.

2. POLARIMETRIC SAR DATA STATISTICS

A polarimetric SAR measures the scattering vector, i.e. the Sinclair matrix represented in vector form, which is defined as

$$\boldsymbol{s} = [s_{hh}\ s_{hv}\ s_{vh}\ s_{vv}]'\ , \tag{1}$$

where $[\cdot]'$ denotes transposition and the s_{bq} are complex scattering coefficients subscripted with transmit polarization b and receive polarization q, that can be horizontal (h) or vertical (v).

The scattering vector $\boldsymbol{s}$ represents the quad-pol SLC measurement. We assume that vector samples $\{\boldsymbol{s}_i\}_{i=1}^{n}$ of size n obtained in local neighborhoods of the sea can be modeled as independent and identically distributed (IID). Introduced by Yueh et al. [2,3], the multivariate product model decomposes the scattering vector as

$$\boldsymbol{s} = \sqrt{\tau}\boldsymbol{x}\ , \tag{2}$$

where the texture τ representing spatial variability of the reflectivity is a real positive random variable with unit mean, $E\{\tau\} = 1$, and probability density function (pdf) $p_\tau(\tau)$, while speckle is represented as a d-dimensional circular complex Gaussian vector, $\boldsymbol{x} \sim \mathcal{N}_d^{\mathbb{C}}(0, \Sigma)$, with zero mean and covariance matrix Σ, which is independent of τ. When the texture variable is assumed to be gamma distributed, the scattering vector becomes multivariate K-distributed.

3. COVARIANCE MATRIX ESTIMATION

Three existing covariance matrix estimators are discussed in this study: the sample mean, fixed-point, and maximum likelihood estimators. An approximate maximum likelihood covariance matrix estimator is also proposed, which provides better computational efficiency.

3.1. Sample mean covariance matrix estimator

The sample mean (SM) covariance matrix estimator is the computationally simplest and it is defined as

$$\hat{\boldsymbol{C}}_{SM} = \frac{1}{n}\sum_{k=1}^{n} \boldsymbol{s}_k \boldsymbol{s}_k^\dagger\ , \tag{3}$$

where n is the number of samples used for estimation and $\dagger$ is the complex conjugate transpose operator. In many studies, the scattering vector is treated as Gaussian distributed by assuming a constant texture variable in the product model (2). In that case, the sample mean covariance matrix estimator is the maximum likelihood estimator of the covariance matrix, and known to be complex Wishart distributed.

3.2. Fixed-point covariance matrix estimator

The fixed-point (FP) covariance matrix estimator, introduced by Gini and Greco [8] and Conte et al. [9], is defined as

$$\hat{\boldsymbol{C}}_{FP}(i+1) = \frac{1}{n}\sum_{k=1}^{n} \frac{d}{\boldsymbol{s}_k^\dagger \hat{\boldsymbol{C}}_{FP}^{-1}(i)\boldsymbol{s}_k} \cdot \boldsymbol{s}_k \boldsymbol{s}_k^\dagger\ , \tag{4}$$

for $i = 0, 1, 2, \ldots, N$, where N is the number of iterations, d is the dimension of the scattering vector. The estimator converges quickly (less than 10 iterations for practical purposes) under low texture conditions, but takes more iterations under higher texture conditions. The biggest advantage of applying the fixed-point covariance matrix estimator is that it does not require any prior information about the pdf of the texture [10]. The pdf of the fixed-point covariance matrix estimator is known to be asymptotically complex Wishart distributed also when the scattering vector has a non-Gaussian distribution [11,12].

3.3. Maximum likelihood covariance matrix estimator

The maximum likelihood (ML) covariance matrix estimator derived by Raghavan and Pulsone [13,14] is given as

$$\begin{aligned}\hat{\boldsymbol{C}}_{ML}(i+1) &= \frac{1}{n}\sum_{k=1}^{n} \frac{h_{d+1}(\boldsymbol{s}_k^\dagger \hat{\boldsymbol{C}}_{ML}^{-1}(i)\boldsymbol{s}_k)}{h_d(\boldsymbol{s}_k^\dagger \hat{\boldsymbol{C}}_{ML}^{-1}(i)\boldsymbol{s}_k)} \cdot \boldsymbol{s}_k \boldsymbol{s}_k^\dagger \\ &= \frac{1}{n}\sum_{k=1}^{n} c_d(\boldsymbol{s}_k^\dagger \hat{\boldsymbol{C}}_{ML}^{-1}(i)\boldsymbol{s}_k) \cdot \boldsymbol{s}_k \boldsymbol{s}_k^\dagger\ ,\end{aligned} \tag{5}$$

for $i = 0, 1, 2, \ldots, N$, where N is the number of iterations, d is the dimension of the scattering vector, and $c_d(t)$ is defined as $h_{d+1}(t)/h_d(t)$. The function $h_d(t)$ is defined as

$$h_d(t) = \int_0^{+\infty} \tau^{-d} \exp\left(-\frac{t}{\tau}\right) p_\tau(\tau) d\tau\ . \tag{6}$$

The convergence in terms of the numbers of iterations of the maximum likelihood covariance matrix estimator is similar to the fixed-point estimator, but each iteration takes longer time. Note that the shape parameter estimation for the texture pdf is unavoidable. Hence, we have a compound estimation problem. The distribution of the maximum likelihood covariance matrix estimator is unknown.

3.4. Approximate maximum likelihood covariance matrix estimator

Here we propose an approximation of the maximum likelihood covariance matrix estimator. The pdf of a unit mean gamma distributed texture is

$$p_\tau(\tau) = \frac{1}{\Gamma(\alpha)}(\alpha)^\alpha \tau^{\alpha-1}\exp(-\alpha\tau) \ , \tag{7}$$

where α is the shape parameter (the larger the shape parameter, the lower the texture). Therefore, the coefficients $c_d(t)$ of the maximum likelihood covariance matrix estimator in (5) can be derived as

$$c_d(t) = \sqrt{\frac{\alpha}{t}}\frac{K_{\alpha-d-1}(\sqrt{4\alpha t})}{K_{\alpha-d}(\sqrt{4\alpha t})} \ , \tag{8}$$

where d is the dimension of the scattering vector, $K_\nu(z)$ is the modified Bessel function of the second kind with the order parameter $\nu = \alpha - d - 1$ or $\alpha - d$.

In [7], Segura obtained the lower and upper bounds for the ratio of Bessel K functions given as

$$0 < L_\nu^{(0)}(z) < \frac{1}{z}\frac{K_\nu(z)}{K_{\nu+1}(z)} < U_\nu^{(0)}(z) \ . \tag{9}$$

The lower bound $L_\nu^{(0)}(z)$ and the upper bounds $U_\nu^{(0)}(z)$ are defined as

$$\begin{aligned} L_\nu^{(0)}(z) &= \left(\nu + 1/2 + \sqrt{(\nu+1/2)^2 + z^2}\right)^{-1} , \\ U_\nu^{(0)}(z) &= \left(\nu + \sqrt{\nu^2 + z^2}\right)^{-1} , \end{aligned}$$

but the upper bound is only sharp when $\nu > 0$, which translates into $\alpha > d + 1$ for the ratio in (8). The lower and upper bounds can be iteratively refined, although the range of applicability, with respect to ν, decreases with the number of iterations. In each iteration, the following sequences are considered

$$\begin{aligned} L_\nu^{(j+1)}(z) &= \frac{1}{2\nu + z^2 U_{\nu-1}^{(j)}(z)} , \\ U_\nu^{(j+1)}(z) &= \frac{1}{2\nu + z^2 L_{\nu-1}^{(j)}(z)} . \end{aligned}$$

For full details, see section 3.2 in [7].

Using the mean value of the lower and upper bounds for the Bessel K function ratio, the coefficients $c_d(t)$ in (8) can be approximated. However, when the order parameter ν is smaller than zero, only the lower bound will hold. Therefore, for any shape parameter $\alpha < d + 1$, the lower bound is used as the approximation, instead of the mean value. Hence, the approximate maximum likelihood (AML) covariance matrix estimator can be given as

$$\hat{\boldsymbol{C}}_{AML}(i+1) = \frac{1}{n}\sum_{k=1}^{n} \hat{c}_d(\boldsymbol{s}_k^\dagger \hat{\boldsymbol{C}}_{AML}^{-1}(i)\boldsymbol{s}_k) \cdot \boldsymbol{s}_k \boldsymbol{s}_k^\dagger \ , \tag{10}$$

for $i = 0, 1, 2, \ldots, N$, where N is the number of iterations, $\hat{c}_d$ are the coefficients using the approximation for the Bessel K function ratio. This approximate estimator converges at a similar rate as the exact version, and takes shorter time for each iteration.

4. RESULTS AND DISCUSSIONS

4.1. Kullback-Leibler matrix distances

In this study, the Kullback-Leibler (KL) matrix distance is used for the comparison of different covariance matrix estimators. It is defined as

$$D_{KL} = \frac{tr(\boldsymbol{C}^{-1}\hat{\boldsymbol{C}}) + tr(\hat{\boldsymbol{C}}^{-1}\boldsymbol{C})}{2} - d \ , \tag{11}$$

where $tr(\cdot)$ is the trace perator, $\hat{\boldsymbol{C}}$ is the estimated covariance matrix, and $\boldsymbol{C}$ is the population mean covariance matrix used in the data simulations. Monte Carlo simulations are performed under various texture conditions with specified shape parameters. Large shape parameters correspond to low texture conditions, and vice versa. Note that the shape parameters used in the maximum likelihood and approximate maximum likelihood covariance matrix estimators are set to be the same as the specified shape parameters for simulations, to eliminate the effects due to the imperfect shape parameter estimation.

Figure 1 shows the experimental results from the covariance matrix estimation comparison of the sample mean (dotted line), fixed-point (dashed line), and maximum likelihood (solid line) estimators. In general, more samples used in the estimations leads to better estimation accuracy (smaller KL matrix distances) as we expect. The maximum likelihood covariance matrix estimator provides the best estimation results under all texture conditions. The sample mean covariance matrix estimator shows the largest estimation error under high texture conditions, but it approaches the maximum likelihood under lower texture conditions. The fixed-point covariance matrix estimation results are better than the sample mean under high texture conditions, but under lower texture conditions, it provides less accuracy and does not come close to the maximum likelihood value. The findings also confirm a previous theoretical study by Gini and Greco [8].

In Figure 2, the proposed approximate maximum likelihood covariance matrix estimator (dashed line) is compared with the exact maximum likelihood covariance matrix estimator (solid line). Under low texture conditions, the approximations are very close to the exact version. However, with higher texture conditions, more and more variations begin to appear. That is because of the limitation for calculating the refined lower and upper bounds for the Bessel K function ratio.

As we can see from the results, in term of the KL matrix distance (estimation accuracy), the maximum likelihood is the best choice through all texture conditions. The sample mean and approximate maximum likelihood covariance matrix estimators are good alternatives under low texture conditions. However, the fixed-point covariance matrix estimator only produces reasonable results under high texture conditions, e.g., high sea states.

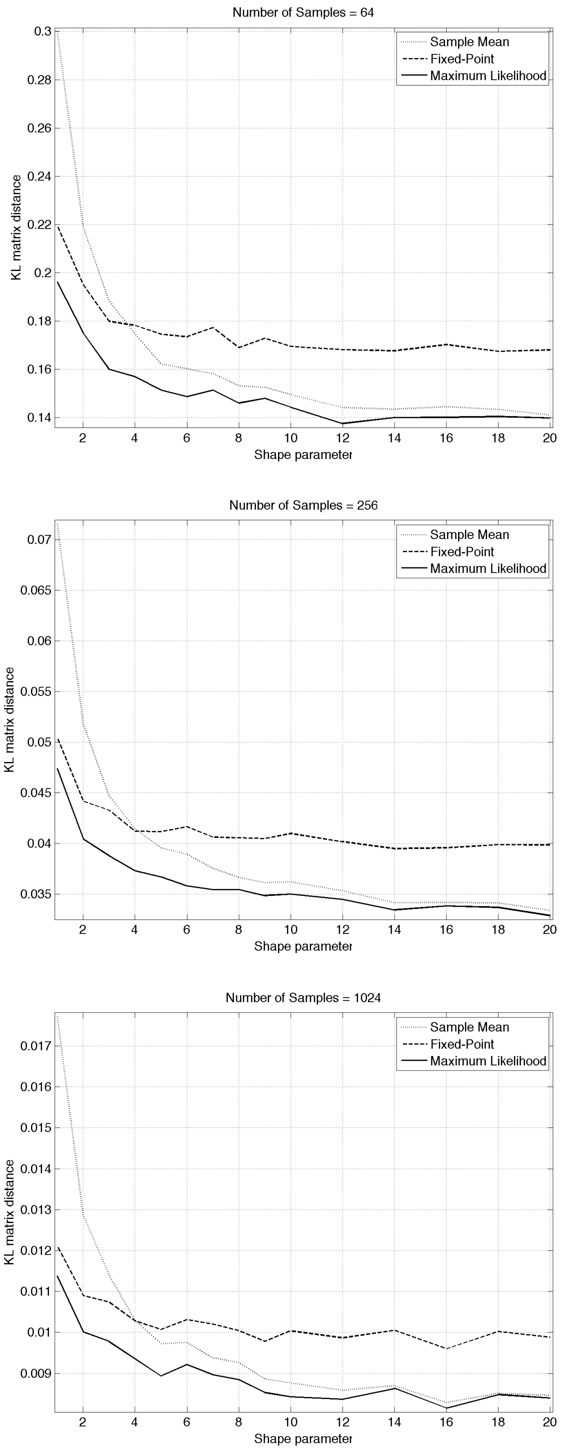

Figure 1. Covariance matrix estimation comparison of three existing estimators at different specified shape parameters with 64, 256 and 1024 samples. The KL matrix distance is applied as a distance measurement between the specified and the estimated covariance matrix. Each data point represents the mean value of 1000 repetitions.

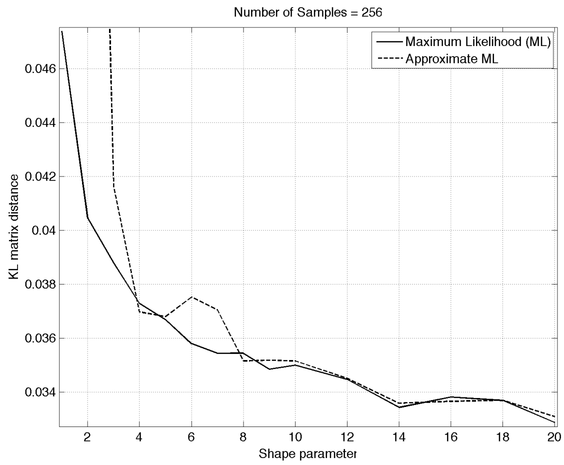

Figure 2. Covariance matrix estimation comparison between the exact and approximate maximum likelihood estimators at different specified shape parameters with 256 samples. The KL matrix distance is applied as a distance measurement between the specified and the estimated covariance matrix. Each data point represents the mean value of 1000 repetitions.

4.2. Computational efficiency and complexity

Section 3 provides a general idea of the computational complexity of different covariance matrix estimators. The well-known sample mean covariance matrix estimator is the simplest, while the others usually require iterative computations. The exact maximum likelihood covariance matrix estimator is the most sophisticated, which contains evaluation of special functions with high computational cost.

Table 1. Computation time comparison for covariance matrix estimation at different specified shape parameters α, with 256 samples. Each value represents the mean of 1000 repetitions. Units in milliseconds.

Textures	α	$\hat{C}_{SM}$	$\hat{C}_{FP}$	$\hat{C}_{ML}$	$\hat{C}_{AML}$
high	1	1.36	6.24	24128.71	77.97
moderate	5	1.36	5.96	7935.55	17.90
low	10	1.37	5.80	5421.32	13.22
very low	20	1.38	5.69	5072.61	9.99

Table 1 shows the computation time comparison for covariance matrix estimation at different specified shape parameters with 256 samples (units in milliseconds). Generally, higher texture conditions lead to more iterations and longer computation time. The maximum likelihood covariance matrix estimator is a lot more time-consuming than the sample mean and fixed-point estimators. By applying the proposed approximation, the computational efficiency is increased significantly. The approximate maximum likelihood covariance matrix estimator takes only about 1% of the computation time of the exact version.

5. CONCLUSIONS

Ship detection based on polarimetric SAR imagery and CFAR detectors requires accurate statistical modeling of ocean clutter. This study addresses the problem of covariance matrix estimation for ocean clutter modeling. Three existing covariance matrix estimators, i.e., the sample mean, fixed-point, and maximum likelihood estimators, are investigated and compared with simulated clutter under various texture conditions. From the experimental results, larger estimation sample sizes and lower texture conditions usually lead to better estimation accuracy, while higher texture conditions require longer computation time. The sample mean and fixed-point covariance matrix estimators perform well under low texture and high texture conditions, respectively. Both of them have good computational efficiency. The maximum likelihood covariance matrix estimator provides the most accurate estimation results under all texture conditions, however, it requires prior knowledge of the pdf of the texture in the product model and long computation time. Hence, an approximate maximum likelihood covariance matrix estimator is proposed, which has good estimation accuracy under low texture conditions and reduces the computation time about 100 times compared to the exact version. Note that the maximum likelihood covariance matrix estimator and its approximation require shape parameter estimation, but that is not in the scope of this article.

In practical applications, estimation sample size, computational efficiency and estimation accuracy are very common trade-offs. Larger estimation sample sizes increase the risk of having heterogeneous samples with non-stationary statistics. A combination of the sample mean and the fixed-point covariance matrix estimators can be considered as an alternative for the maximum likelihood with good accuracy and computational efficiency. The distribution functions of the covariance matrix estimators are also important for the subsequent processing. For instance, the sample mean covariance matrix estimator is known to be complex Wishart distributed under the assumption of having Gaussian distributed scattering vector; the fixed-point covariance matrix estimator is known to be asymptotically complex Wishart distributed with non-Gaussian clutter; and the maximum likelihood covariance matrix estimator distribution function is unknown to the authors knowledge.

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