ESTIMATION OF THE EQUIVALENT NUMBER OF LOOKS IN POLARIMETRIC SAR IMAGERY

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ABSTRACT

We present two new estimators for the equivalent number number of looks (ENL) designed for polarimetric synthetic aperture radar data. These are the first known estimators to utilise the full multilook polarimetric covariance matrix, and both are derived from moments of the Wishart distribution. The first estimator is obtained from the second-order trace moment, which provides a multivariate generalisation of the conventional ENL definition. The second estimator is found from the log-determinant moment, and is also shown to be the maximum likelihood estimator. The latter estimator proves to have superior statistical properties to any other known ENL estimator. These properties are decisive for the good performance obtained with a proposed procedure for unsupervised estimation. We demonstrate on airborne AIRSAR data how a robust ENL estimate can be extracted from the distribution of small sample estimates collected over the whole scene.

Index Terms**—** Parameter estimation, statistical modelling, polarimetric synthetic aperture radar.

1. INTRODUCTION

The equivalent (or effective) number of looks (ENL) is a parameter of multilook synthetic aperture radar (SAR) images, which describes the degree of averaging applied to the SAR measurements during data formation and sometimes also postprocessing. Multilooking is performed in order to mitigate the noiselike effect of interference, known as speckle, which is characteristic of all coherent imaging systems [1].

The multilooking consists of averaging single-look images, obtained by splitting the synthetic aperture Doppler bandwidth into subbands. These so-called looks are correlated, which largely complicates exact statistical modelling of the result [1]. The pragmatic solution is to model multilook data as an average of independent measurements, and to replace the actual number of correlated samples by an equivalent number of independent ones, i.e., the ENL. The ENL estimate is the generally non-integer parameter value that makes certain moment relations of the theoretical model consistent with empirical moments.

The ENL is a parameter of various distributions used to model multilook SAR data. Hence, it has influence on the accuracy of the information extracted by methods based upon statistical modelling. For instance, the ENL is necessary input to important classification and change detection algorithms for polarimetric SAR (PolSAR) data.

Estimation is commonly done by identifying homogeneous regions in an image, where the contribution of texture is negligible, meaning that the radar cross section is assumed to be constant. These conditions assure that the distribution of the scattering coefficients can be assumed complex Gaussian, and the ENL can be estimated from simple image statistics [1]. Underestimation occurs in the presence of texture and other sources of inhomogeneity.

The ENL and its conventional estimator have been defined for the case of single polarisation SAR [1]. For PolSAR data, it has traditionally been estimated separately for each polarimetric channel, and then averaged, as in [2]. Our first objective is to extend the theory of ENL estimation to the polarimetric case, where estimates are derived explicitly from multivariate statistics.

A reliable ENL estimate is not always provided with the data set, or it may be altered by postprocessing performed by the end user. Moreover, the identification of a homogeneous region where image statistics can be calculated, may not be trivial. Our second goal is therefore to design a fully automatic estimation procedure that requires no parameter selection or manual region extraction.

In Sec. 2 we present background theory on polarimetric SAR, data models, and previous work on ENL estimation. In Sec. 3 we propose two new ENL estimators and a procedure for unsupervised estimation. The results are presented and discussed in Sec. 4, before conclusions are given in Sec. 5.

2. THEORY

The full-polarimetric SAR instrument measures the complete scattering matrix:

$$\mathbf{S} = \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix}, \tag{1}$$

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where the complex scattering coefficients, $S_{rt};\ r,t \in \{h,v\}$, describe the transformation of transmitted to received electromagnetic field for all combinations of horizontal (h) and vertical (v) polarisation at the receiver (r) and transmitter (t).

For monostatic radar and reciprocal media, the scattering matrix can be vectorised as

$$\mathbf{s} = \begin{bmatrix} S_{hh} \\ (S_{hv}+S_{vh})/\sqrt{2} \\ S_{vv} \end{bmatrix} \text{ or } \mathbf{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{hh}+S_{vv} \\ S_{hh}-S_{vv} \\ S_{hv}+S_{vh} \end{bmatrix}, \tag{2}$$

where $\mathbf{s}$ is known as the lexicographic scattering vector. The Pauli basis scattering vector, denoted $\mathbf{k}$, is a linear transformation of $\mathbf{s}$, which provides physical interpretations of its elements in terms of basic scattering mechanisms. The scaling with a factor $\sqrt{2}$ is done to preserve total power of the signal. Assume that $\mathbf{s}$ (or $\mathbf{k}$) is circular complex multivariate Gaussian with mean vector $\mathrm{E}\{\mathbf{s}\} = \mathbf{0}$, covariance matrix $\mathbf{\Sigma_s} = \mathrm{E}\{\mathbf{ss}^H\}$, and dimension d. The superscript H denotes conjugate transpose. This is a common simplification, although it is strictly justified only for homogeneous regions of the image, characterised by fully developed speckle and no texture, i.e., no spatial variation in the backscatter due to fluctuations in the radar cross section.

Multilook PolSAR data is normally represented by

$$\mathbf{C_s} = \frac{1}{L}\sum_{i=1}^{L} \mathbf{s}_i\mathbf{s}_i^H \text{ or } \mathbf{C_k} = \frac{1}{L}\sum_{i=1}^{L} \mathbf{k}_i\mathbf{k}_i^H, \tag{3}$$

known as the target covariance matrix and coherency matrix, computed from the single-look scattering vectors $\{\mathbf{s}_i\}_{i=1}^L$ and $\{\mathbf{k}_i\}_{i=1}^L$, respectively. L is the nominal number of looks. It follows from the Gaussian assumption that if $L \geq d$ and the $\mathbf{s}_i$ (or $\mathbf{k}_i$) in (3) are independent, then the scaled covariance matrix, defined as $\mathbf{Z} = L\mathbf{C_s}$ (or $\mathbf{Z} = L\mathbf{C_k}$), is complex Wishart distributed with probability density function (pdf) [3]

$$p_{\mathbf{Z}}(\mathbf{Z}; L, \mathbf{\Sigma}) = \frac{|\mathbf{Z}|^{L-d}}{|\mathbf{\Sigma}|^L \Gamma_d(L)} \exp\left(-\operatorname{tr}\left(\mathbf{\Sigma}^{-1}\mathbf{Z}\right)\right), \tag{4}$$

where $\operatorname{tr}(\cdot)$ is the trace operator, $|\cdot|$ is the determinant, and $\mathbf{\Sigma} = \mathrm{E}\{\mathbf{Z}\}/L = \mathrm{E}\{\mathbf{C_s}\}$. The normalisation constant

$$\Gamma_d(L) = \pi^{d(d-1)/2} \prod_{i=0}^{d-1} \Gamma(L-i), \tag{5}$$

is a multivariate extension of the gamma function, $\Gamma(L)$.

The distribution of $\mathbf{C_s}$, which is a linear transform of (4), reduces to the gamma distribution for $d = 1$. The pdf of a single polarisation intensity, denoted I, is thus

$$p_I(I; \sigma, L) = \frac{1}{\Gamma(L)} \left(\frac{L}{\sigma}\right)^L I^{L-1} e^{-LI/\sigma}, \tag{6}$$

where the mean intensity σ and the number of looks L are the parameters. The r-th order moment of I is given by [1]

$$\mathrm{E}\{I^r\} = \frac{\Gamma(L+r)}{\Gamma(L)} \left(\frac{\sigma}{L}\right)^r, \tag{7}$$

assuming uncorrelated data. We specifically find that $\mathrm{E}\{I\} = \sigma$ and $\mathrm{Var}\{I\} = \sigma^2/L$, thus $\mathrm{E}\{I\}^2/\mathrm{Var}\{I\} = L$. This does not hold for correlated data, but in this case L can be replaced by the ENL, defined as

$$L_e = \frac{E\{I\}^2}{\mathrm{Var}\{I\}}. \tag{8}$$

The conventional (CV) estimator of L_e thus arises as

$$\hat{L}_e^{(CV)} = \frac{\langle I \rangle^2}{\langle I^2 \rangle - \langle I \rangle^2}, \tag{9}$$

where $\langle \cdot \rangle$ denotes sample average.

From the sparse literature on ENL estimation, we highlight an alternative estimator presented by Frery et al. in [2]. From (7), we have the fractional moment (FM)

$$\mathrm{E}\{I^{1/2}\} = \frac{\Gamma(L+\frac{1}{2})}{\Gamma(L)} \sqrt{\frac{\sigma}{L}}, \tag{10}$$

which can also be used to solve for L. Replacing $\mathrm{E}\{I^{1/2}\}$ and σ with the estimates $\langle I^{1/2} \rangle$ and $\langle I \rangle$, we obtain

$$f\big(\hat{L}_e^{(FM)}\big) = \frac{\Gamma\big(\hat{L}_e^{(FM)} + \frac{1}{2}\big)}{\Gamma\big(\hat{L}_e^{(FM)}\big)\sqrt{\hat{L}_e^{(FM)}}} \sqrt{\langle I \rangle} - \langle \sqrt{I} \rangle = 0, \tag{11}$$

which must be solved numerically for the fractional moment estimate, denoted $\hat{L}_e^{(FM)}$. This method was applied to polarimetric SAR data in [2], estimating the ENL separately for each polarisation, and then averaging the results.

3. NEW ESTIMATORS

We have not found any ENL estimators that use the full target covariance or coherency matrix, or any other multivariate statistic, and thereby utilise all the available statistical information. In an attempt to derive moment based estimators founded on the Wishart distribution, we use the second-order trace moment of $\mathbf{Z}$, derived in [4] as

$$\mathrm{E}\{\operatorname{tr}(\mathbf{ZZ})\} = \operatorname{tr}(\mathbf{\Sigma\Sigma}) + \frac{1}{L}\operatorname{tr}(\mathbf{\Sigma})^2. \tag{12}$$

This leads to the trace moment (TM) estimator for L_e:

$$\hat{L}_e^{(TM)} = \frac{\operatorname{tr}(\mathbf{\Sigma})^2}{\langle \operatorname{tr}(\mathbf{ZZ}) \rangle - \operatorname{tr}(\mathbf{\Sigma\Sigma})}. \tag{13}$$

We note that it uses all the elements of $\mathbf{Z}$ through $\operatorname{tr}(\mathbf{ZZ})$. Eq. (13) is invariant to scaling of $\mathbf{Z}$. Moreover, it reduces to (9) in the single polarisation case. We have thus found a multivariate generalisation of the conventional ENL estimator, and, when $\langle \cdot \rangle$ is replaced by $\mathrm{E}\{\cdot\}$, also of the definition in (8).

For our second estimator, we start from the moments of the normalised determinant of a Wishart matrix, given as

$$\mathrm{E}\left\{\left(\frac{|\mathbf{Z}|}{|\mathbf{\Sigma}|}\right)^r\right\} = \prod_{i=0}^{d-1} \frac{\Gamma(L-i+r)}{\Gamma(L-i)} \tag{14}$$

by extension of a result in [5] from real to complex analysis. This expression is identified as the moment-generating function of the normalised log-determinant, whose moments follow after some calculus. Specifically, the first-order log-determinant moment becomes

$$\mathrm{E}\left\{\ln\left(\frac{|\mathbf{Z}|}{|\mathbf{\Sigma}|}\right)\right\} = \sum_{i=0}^{d-1}\Psi^{(0)}(L-i)\,, \tag{15}$$

where $\Psi^{(0)}(L) = \Gamma'(L)/\Gamma(L)$ is the digamma function. Details of the derivation are omitted due to limited space. Note that for a sample of independent and identically distributed Wishart matrices, $\mathscr{Z} = \{\mathbf{Z}_1, \ldots, \mathbf{Z}_n\}$, we have from (4):

$$\begin{aligned}\frac{\partial}{\partial L}&\ln p_{\mathbf{Z}}(\mathscr{Z}; L, \mathbf{\Sigma})\\ &= n\left(\frac{1}{n}\sum_{i=1}^{n}\ln|\mathbf{Z}_i| - \ln|\mathbf{\Sigma}| - \frac{\partial}{\partial L}\ln\Gamma_d(L)\right)\\ &= n\left(\langle\ln|\mathbf{Z}_i|\rangle - \ln|\mathbf{\Sigma}| - \sum_{i=0}^{d-1}\Psi^{(0)}(L-i)\right).\end{aligned} \tag{16}$$

By comparison with (15), we see the equivalence of the maximum likelihood (ML) solution, defined by $\partial\ln p_{\mathbf{Z}}(\mathscr{Z})/\partial L = 0$, and the estimator based on the log-determinant moment, which is thus asympotically unbiased, efficient, and Gaussian.

Because data is supplied as matrices in the format $\mathbf{C} = \mathbf{Z}/L$, we substitute $\ln|\mathbf{Z}| = \ln|\mathbf{C}| + d\ln L$. After the mathematical expectation $\mathrm{E}\{\ln|\mathbf{C}|\}$ has been replaced by $\langle\ln|\mathbf{C}|\rangle$ and $\mathbf{\Sigma}$ by $\langle\mathbf{C}\rangle$, we have the equation

$$\begin{aligned}g\big(\hat{L}_e^{(ML)}\big) &= \langle\ln|\mathbf{C}|\rangle - \ln|\langle\mathbf{C}\rangle|\\ &\quad - \sum_{i=0}^{d-1}\Psi^{(0)}\big(\hat{L}_e^{(ML)} - i\big) + d\ln\hat{L}_e^{(ML)} = 0\,,\end{aligned} \tag{17}$$

which defines the ML estimator, denoted $\hat{L}_e^{(ML)}$. The root of (17) must be found numerically in the same way as for (11).

As an approach to unsupervised estimation, we propose to calculate small sample ENL estimates in a sliding window covering the whole SAR scene, a method inspired by [6] and [7]. From the collection of local estimates, we compute a nonparametric kernel density estimate (KDE) and extract the global estimate from the mode. The local samples are not tested for homogeneity and conformity to the Wishart model, as suggested in [2]. We simply assume that the ENL estimates obtained under the required statistical assumptions will dominate the density estimate, and that effects of texture and mixed classes within the estimation window are negligible. With small sample sizes, bias reduction becomes vital, and this is achieved by using jackknife estimates of the bias [8].

4. RESULTS AND DISCUSSION

In the experiments we used synthetic and real data to compare the FM, TM and ML estimator. We first tested the statistical properties of the estimators on random generated data from a single Wishart distributed class, with $L = 10$ and the parameter matrix $\mathbf{\Sigma}$ estimated from a region of the NASA/JPL AIRSAR L-band image of Flevoland, the Netherlands. This 4-look data set was also used to test the unsupervised estimation procedure. No speckle filter was applied.

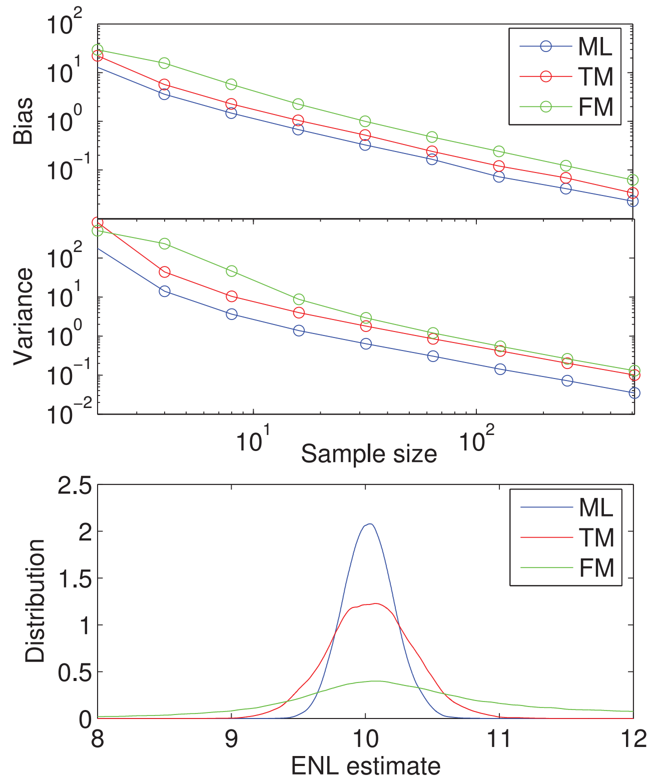

Fig. 1. Estimator bias and variance as a function of sample size n, and the distribution of ENL estimates for $n = 512$.

Fig. 1 shows bias (upper panel) and variance (middle panel) versus sample size for the three estimators. These are bootstrap estimates [8] obtained from $M = 10{,}000$ bootstrap samples of variable sample size n, drawn from a population of $N = 1{,}000{,}000$ random generated coherency matrices. The lower panel shows the distribution of ENL estimates for a fixed sample size of $n = 512$. It was computed with a KDE estimator with Epanechnikov kernel and kernel size $h = 0.1$. We observe that the TM estimator ranks better than the FM estimator, but that the ML estimator is clearly the superior.

The strong statistical properties of the ML estimator are also demonstrated in Fig. 2. The five upper panels show densities of the local small sample estimates for all the different estimators and for different sizes $k \times k$ of the estimation window in the unsupervised estimation procedure. Only the ML estimator produces a density with a significant mode that can be related to the true ENL value, which is known from earlier studies to be a little greater than 3. Certain features of the distribution of ML estimates are observed for increasing k: The width of the peak decreases, as expected; A second mode emerges at around 2.7, which represents the increasing number of estimation windows containing mixed class pixels; The mode value experiences a negative shift. The last point is explained by the estimator bias. The lowest panel visualises the variation of the mode value with k. It shows that the decreasing trend disappears when the bias correction is applied. Note that the jackknife bias estimate itself becomes biased when the sample size is very small, which explains the strange behaviour of the bias corrected estimate for the lowest value of k. The influence of texture has not been determined.

We have observed that the mode estimation is robust with respect to the choice of kernel size, and simplified selection rules can be applied. A window size of $k = 5$ or 7 seems to provide a good compromise between maintaining homogeneity and avoiding bias problems. With these decisions handled, a fully unsupervised estimation procedure has been specified. It yields an ENL estimate of $\hat{L}_e = 3.20$ for the Flevoland image. Further testing on data from different sources is required, but the results are promising.

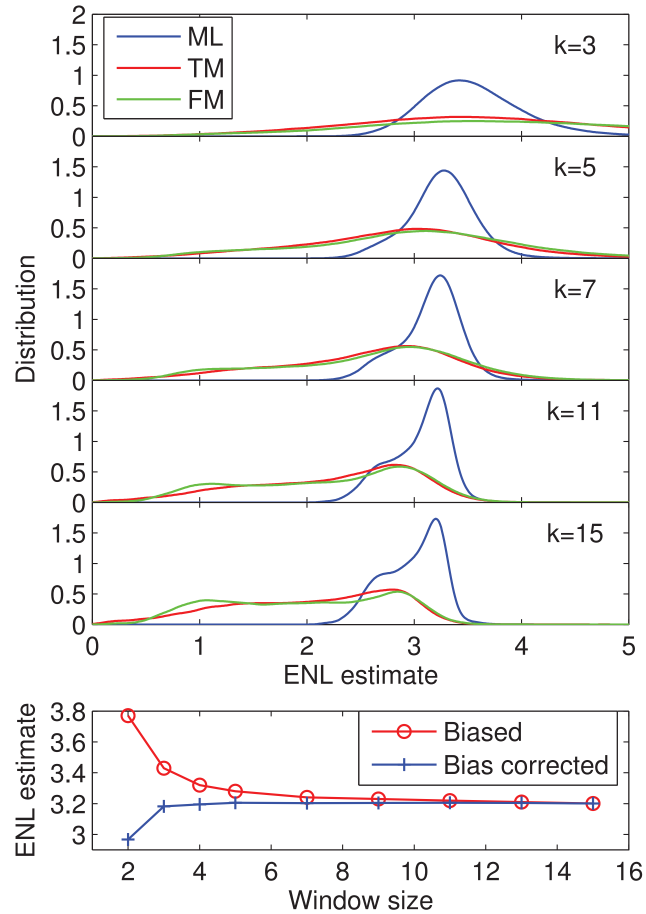

Fig. 2. Distributions of local ENL estimates from the Flevoland image for different values of the window size k (upper panel). Global ENL estimates versus k (lowest panel).

5. CONCLUSIONS

We have proposed a generalised definition of the ENL that accomodates polarimetric SAR data. It was derived from the second-order trace moment of a Wishart distributed matrix, and is associated with a closed-form ENL estimator that uses the full target covariance or target coherency matrix as input. A second estimator was obtained from the log-determinant moment of a Wishart matrix, which also utilises all the available statistical information. This estimator is a nonclosed-form expression, which must be solved numerically, but posesses superior statistical properties compared to all known ENL estimators. This has been documented by experimental results. It was also shown to be the maximum likelihood (ML) estimator under the Wishart model. We have finally proposed an unsupervised estimation procedure that utilises the strength of the ML estimator, and demonstrated its capabilities on airborne full-polarimetric AIRSAR data.

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